Reg.No. \_\_\_\_\_\_\_\_\_\_\_\_



**UNIVERSITY**

(Karunya Institute of Technology & Sciences)

(Declared as Deemed-to-be University under Sec.3 of the UGC Act, 1956)

**End Semester Examination – Nov/Dec – 2016**

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|  |  | **Semester :** | **2016-17 ODD** |
| **Code :** | **14MA3005** | **Duration :** | **3hrs** |
| **Sub. Name :** | **Calculus of Variations and Vector Spaces** | **Max. marks :** | **100** |

**ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)**

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| **Q. No.** | **Sub Div.** | **Questions** | **Course**  **Outcome** | **Marks** |
| 1. | a. | State and prove Brachisto Chrone problem. | CO1 | 12 |
| b. | Find the extremal of the functional | CO1 | 8 |
| (OR) | | | | |
| 2. | a. | Find the plane curve of fixed perimeter and maximum area. | CO1 | 10 |
| b. | Find the extremal of the functional  given that y(0) = 0, y’(0)=1, y(1) = 1, y’(1) = 1. | CO1 | 10 |
| 3. | a. | Find the differential equation corresponding to the integral equation | CO2 | 10 |
|  | b. | Find the integral equation corresponding to the differential equation , y(0) = 0, y’(0) = 1. | CO2 | 10 |
| (OR) | | | | |
| 4. | a. | Using the method of successive approximations, solve the integral equation | CO2 | 10 |
|  | b. | Find the characteristic numbers and eigen functions for the homogeneous integral equation | CO2 | 10 |
| 5. | a. | Prove that union of two subspaces is a subspace if and only if one is contained in the other. | CO3 | 10 |
|  | b. | Prove that union of two subspaces need not be a subspace. | CO3 | 3 |
|  | c. | Prove that arbitrary intersection of subspaces is a subspace of a vector space V. | CO3 | 7 |
| (OR) | | | | |
| 6. | a. | Are the vectors α1 = (-1,2,-2), α2 = (1,2,1), α3 = (-1,-2,0) linearly independent in R3? | CO3 | 5 |
|  | b. | Find out values of k for which the set S of vectors (3,1,2), (-2,k,5), (19k,18,19k) is not a basis of R3. | CO3 | 10 |
|  | c. | Verify whether  is a subspace or not. | CO3 | 5 |
| 7. | a. | Let V be an inner product space. Then prove that (i) ||cα|| = |c| ||α||  (ii) ||α|| > 0 for α ≠ 0 (iii)  and (iv) ||α+β|| ≤ ||α|| + ||β|| for α, β V. | CO3 | 20 |
| (OR) | | | | |
| 8. | a. | Let V be the set of all polynomials of degree ≤ 2 together with the zero polynomial. V is a real inner product space with inner product defined by  Starting with the basis {1, x, x2}, obtain an orthonormal basis for V. | CO3 | 10 |
|  | b. | Apply the Gram-Schmidt process to the vectors β1 = (3,0,4), β2 = (-1,0,7), β3 = (2,9,11) to obtain an orthonormal basis of R3 with the standard inner product. | CO3 | 10 |
|  | | **Compulsory:** |  |  |
| 9. | a. | Find the inverse Z-transform of | CO1 | 10 |
|  | b. | Solve  with y0 = 0, y1 = 0 using Z-transform. | CO1 | 10 |

ALL THE BEST